HARTLEY, FOURIER COSINE GENERALIZED CONVOLUTION INEQUALITY

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ABSTRACT. In this paper we construct and study generalized convolutions $(f \underset{1}{*} g)(x))$ of functions f, g for the Hartley (H_1, H_2), Fourier cosine (F_c) integral transform:

$$(f \underset{1}{*} g)(x) = \frac{1}{2\pi} \int_0^\infty [g(x+u) + g(x-u)]\, f(u)\, du, \quad x \in \mathbb{R}, \tag{0.1}$$

whose factorization equalities is of the form:

$$H_{\left\{ \substack{1 \\ 2} \right\}}(f \underset{1}{*} g)(y) = (F_c f)(y)(H_{\left\{ \substack{1 \\ 2} \right\}} g)(y), \quad \forall y \in \mathbb{R}.$$

We obtain the existence of new generalized convolutions on different function spaces, such as $L_1(\mathbb{R}), L_p^{\alpha,\beta,\gamma}(\mathbb{R})(r > 1)$. Besides, we apply new generalized convolutions in order to prove the theorems type Young (0.2) and Inequality type Saitoh (0.3) as follows.

$$\left| \int_0^\infty (f \underset{1}{*} g)(x) h(x) dx \right| (x) \leqslant \frac{\sqrt[p]{2}}{\sqrt{2\pi}} \|f\|_{L_p(\mathbb{R}_+)} \|g\|_{L_q(\mathbb{R})} \|h\|_{L_r(\mathbb{R})}. \tag{0.2}$$

$$\|((F_1\rho_1) \underset{1}{*} (F_2\rho_2))(\rho_1 \underset{1}{*} \rho_2)^{\frac{1}{p}-1}\|_{L_p(\mathbb{R})} \leqslant \sqrt{\frac{2}{\pi}} \|F_1\|_{L_p(\mathbb{R}_+,\rho_1)} \|F_2\|_{L_p(\mathbb{R},\rho_2)}. \tag{0.3}$$

In particular, we prove the Inequality reverse Young and Inequality reverse Saitoh The applications in solving integral equations, integral equations and estimate the solution of some equations are presented.

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